A mixed multi-field representation of gradient-type problems in solid mechanics

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The modeling of phase fields and size effects in solids, such as the width of shear bands or the grain size dependence of the plastic flow in poly-crystals, need to be based on non-standard continuum approaches which incorporate length-scales. With the ongoing trend of miniaturization and nanotechnology, the predictive modeling of these effects play an increasingly important role. The mixed multi-field representation of gradient-type problems is a recently introduced thermomechanically consistent framework for modeling such kind of phenomena [1]. The key idea is to extend the field of constitutive state variables by micromechanical independents and further to derive the macro and micro balance equations in a closed form..

Problem Definition & Objectives

Since the range of application for gradient-type problems is numerous, many specified approaches have been developed throughout the last years. Each of them is specially aligned to the investigated problem and cannot be used in different research fields. In this project a general framework for gradient-type problems in solid mechanics will be derived, in order to provide a modular concept for solving arbitrary gradient-type problems. The concept is implemented into the finite element method by using an efficient and stable mixed finite element technology.

Planned Cooperation

A. Bertram (IFME) and **H. Altenbach** (IFME)
– Gradient plasticity, Packaging processes

T. Halle (IWF) and **M. Krüger** (IWF)
– Solidification processes, Fracture evolution

L. Tobiska (IAN) – FE technology

Generalized form of gradient-type problems

In contrast to classical local approaches with locally evolving internal variables, order parameter fields can be taken into account governed by additional balance-type partial differential equations including micro-structural boundary conditions. This incorporates non-local dissipative effects based on length scales, which reflect properties of the material micro-structure [1].

Field of constitutive state variables

$$\mathbf{g} = \{\mathbf{g}_1, \nabla\mathbf{g}_1, \ldots, \mathbf{g}_2, \nabla\mathbf{g}_2, \ldots\}$$

Stored energy functional

$$\Psi(\mathbf{g}) := \int_{\mathcal{B}} \psi(\mathbf{g})\, dV$$

Dissipation potential functional

$$\Phi(\mathbf{g}, \dot{\mathbf{g}}) := \int_{\mathcal{B}} \phi(\mathbf{g}, \dot{\mathbf{g}})\, dV$$

External power

$$P_{ext}(\dot{\mathbf{g}}) := \int_{\mathcal{B}} b(\dot{\mathbf{g}})\, dV + \int_{\partial\mathcal{B}} t(\dot{\mathbf{g}})\, dA$$

Rate-type potential

$$\Pi(\dot{\mathbf{g}}) := \int_{\mathcal{B}} \underbrace{\frac{d}{dt}\psi(\mathbf{g}) + \phi(\mathbf{g}, \dot{\mathbf{g}})}_{\pi(\mathbf{g},\dot{\mathbf{g}})}\, dV - P_{ext}(\mathbf{g}, \dot{\mathbf{g}}) \;\rightarrow\; 0$$

Macro and micro balance equations

$$\delta_{\dot{\mathbf{g}}_1}\pi \equiv -\mathrm{Div}\left[\partial_{\nabla\mathbf{g}_1}\psi + \partial_{\nabla\dot{\mathbf{g}}_1}\phi\right] = \bar{\mathbf{b}}_1$$

$$\delta_{\dot{\mathbf{g}}_2}\pi \equiv -\mathrm{Div}\left[\partial_{\nabla\mathbf{g}_2}\psi + \partial_{\nabla\dot{\mathbf{g}}_2}\phi\right] + \partial_{\mathbf{g}_2}\psi + \partial_{\dot{\mathbf{g}}_2}\phi = \bar{\mathbf{b}}_2$$

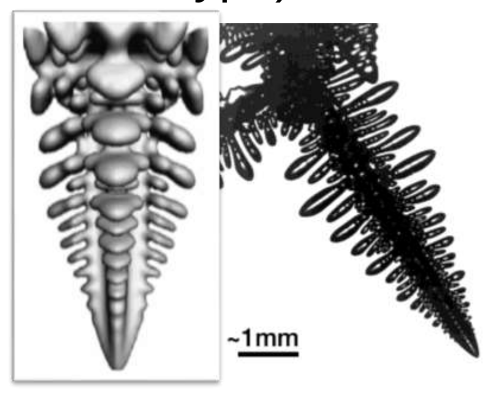
Example 1: Phase transition phenomena (e.g. Ginzburg-Landau-type)

$$\mathbf{g} = \{f, \nabla f\}$$

f : phase concentration

$$\psi = \hat{\psi}(\mathbf{g}) = \psi_s(f) + \psi_i(\nabla f)$$

$$\phi = \hat{\phi}(f, \dot{f})$$



Dendritic growth: simulation [2] and experiment (*mix.msfc.nasa.gov*)

Example 2: Gradient plasticity in single crystals

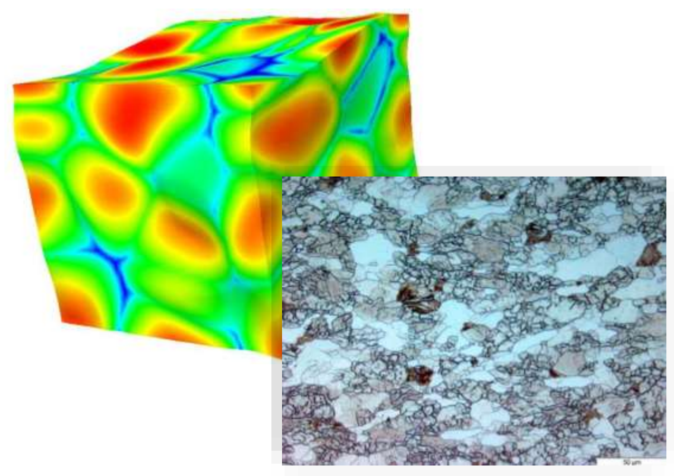
$$\mathbf{g} = \{\mathbf{F}, \gamma, \nabla\gamma\}$$

$\mathbf{F}$: deformation gradient
γ : plastic slip

$$\psi = \hat{\psi}_e(\mathbf{F}, \gamma) + \hat{\psi}_p(\gamma, \nabla\gamma)$$

$$\phi = \hat{\phi}(\mathbf{g}, \dot{\mathbf{g}}, \mathbf{h})$$

$\mathbf{h}$: dual driving forces



Gradient plasticity: simulation and experiment [3]

Mixed finite element technology for multi-field problems

An important aspect for the numerical implementation of gradient-type problems is to design an efficient finite element technology in the full spatial domain, that resolves in a robust format the emergent deficiencies of low-order finite elements in finite deformations (e.g. locking effects in J2-plasticity). For this purpose a mixed finite element technology including linear displacement and pressure interpolations is used and will be extended to multi-field variational problems [4],[5].

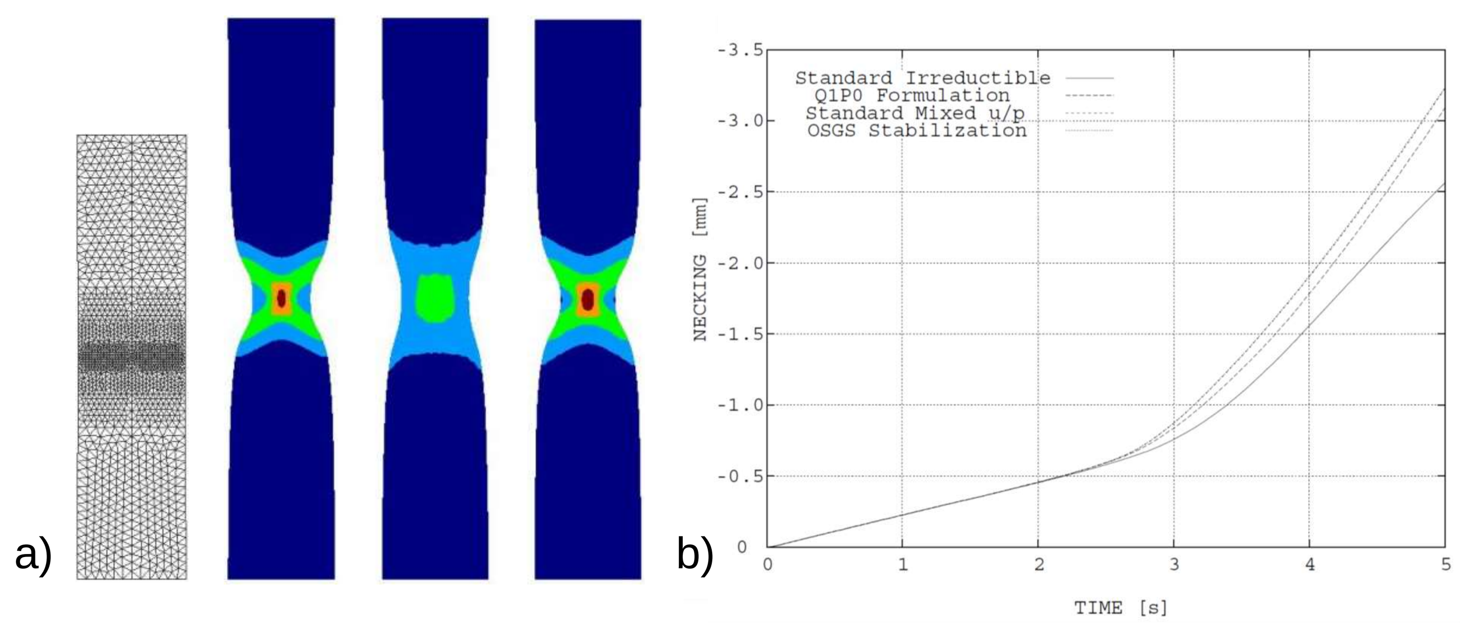
Strong form of mixed formulation

$$J\,\nabla\left(\frac{p}{J}\right) + J\,\nabla\cdot\left(\frac{\mathbf{s}(\mathbf{u})}{J}\right) + \mathbf{f} = \mathbf{0}$$

$$\frac{p}{\kappa} - \ln J - \tau\left(\nabla^2 p - \nabla\cdot\Sigma\right) = 0$$

$$\nabla p - \Sigma = \mathbf{0}$$

τ : stabilization parameter
Σ : pressure gradient projection



Necking of a stretched bar: a) FE discretization and results with Q1P0 / P1 / P1P1 elements, b) Necking displacement

Literature

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[4] D. Christ, M. Cervera, M. Chiumenti, C. Agelet de Saracibar, A mixed finite element formulation for incompressibility using linear displacement and pressure interpolations, Monograph 77, CIMNE (2003)
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